## Principle and Pre-Calculus Math 11 <br> 1.5 Lab - Geometric Series Lab <br> By the end of this lesson I will be able to: <br> - Identify infinite geometric series <br> - Convergent vs. Divergent

## ANSWER KEY

## Part A: Geometric Sequences

1) Choose a positive first term. Choose a common ratio, $r$, in each of the intervals in the table below. For each common ratio, create the first 5 terms of a geometric sequence.

Assume: $\mathrm{t}_{1}=10$

| Interval | Common ratio, $\mathbf{r}$ | Geometric Sequence |
| :---: | :---: | :---: |
| $r>1$ | Example: $r=2$ | $10,20,40,80,160 \ldots$ |
| $0<r<1$ | $r=\frac{1}{2}$ | $10,5,2.5,1.25,0.625 \ldots$ |
| $-1<r<0$ | $r=-\frac{1}{2}$ | $10,-5,2.5,-1.25,0.625 \ldots$ |
| $r<-1$ | $r=-2$ | $10,-20,40,-80,160 \ldots$ |

b) For each SEQUENCE:

Graph the terms numbers on the horizontal axis and the terms values on the vertical axis. Sketch and label each graph on a grid below.

$$
r>1
$$

$0<r<1$
$-1<r<0$

$r<-1$

c) What happens to the term values as more points are plotted for these sequences?

As more points are plotted:

| $r>1$ | They move further from the x-axis; Further away from zero. <br> (Up to the right - increasing) |
| :---: | :---: |
| $0<r<1$ | They move closer to the x-axis; Closer to zero. |
| (Down and to the right - decreasing) |  |

## Part B: Geometric Series

2) Use the four geometric sequences in Part A to create four corresponding geometric series.
a) For each series, complete the table below by calculating these partial sums:

Using $t_{1}=10$, calculate: $S_{1}, S_{2}, S_{3}, S_{4}, S_{5}$

| Interval | Common <br> ratio, $\mathbf{r}$ | $\boldsymbol{S}_{\mathbf{1}}$ | $\boldsymbol{S}_{\mathbf{2}}$ | $\boldsymbol{S}_{\mathbf{3}}$ | $\boldsymbol{S}_{\mathbf{4}}$ | $\boldsymbol{S}_{\mathbf{5}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r>1$ | $r=2$ | 10 | 30 | 70 | 150 | 310 |
| $0<r<1$ | $r=\frac{1}{2}$ | 10 | 15 | $\mathbf{1 7 . 5}$ | 18.75 | 19.375 |
| $-1<r<0$ | $r=-\frac{1}{2}$ | 10 | 5 | 7.5 | 6.25 | 6.875 |
| $r<-1$ | $r=-2$ | 10 | -10 | 30 | -50 | 110 |

b) Graph the numbers of terms in the partial sums on the horizontal axis and the partial sums on the vertical axis. Sketch and label each graph on a grid below.
$r>1$

$-\mathbf{1}<r<\mathbf{0}$

$0<r<1$


$$
\begin{array}{lllll}
S_{1} & S_{2} & S_{3} & S_{4} & S_{5}
\end{array}
$$

$$
r<-1
$$


c) What happens to the term values as more points are plotted for these series?

As more points are plotted:

| $r>1$ | They move further from the x-axis; Further away from zero. (Up to the right - increasing) <br> Divergent |
| :---: | :---: |
| $0<r<1$ | They move closer to a single point (Closer towards $\boldsymbol{x}=\mathbf{2 0}$ ) Convergent |
| $-1<r<0$ | They move closer to a single point (Closer towards $\boldsymbol{x}=6.67$ ) Convergent |
| $r<-1$ | They move further from the $x$-axis; Further away from zero. (Alternating around x -axis - increasing) <br> Divergent |

## New Definitions:

Convergent Series - If the sequence of the partial sums converges to a constant value as the number of terms increases, then the geometric series is convergent.

- The partial sums $\left(S_{n}\right)$ keep getting closer to a constant value so it is called a convergent series.

Divergent Series - If the sequence of the partial sums does not converge to a constant value as the number of terms increases, then the geometric series is divergent.

- The partial sums $\left(S_{n}\right)$ keep getting farther away from a constant value and closer $\infty$ or $-\infty$ so it is called a divergent series.

